[11] H. H. Price, *Perception* (London, 1950). Note, however, that the findings of neurophysiology must be kept in mind when we are evaluating the evidence from neurology and experimental psychology.

[12] Lean, private communication.

[13] Smythies, *loc. cit.*

Axioms for the Part Relation

by NICHOLAS RESCHER

THERE exists a well-known axiomatic theory of the part relation, the *mereology* devised by the Polish logician S. Lesniewski, and subsequently developed by A. Tarski and others.[1] However, as will appear below, this axiom system is not applicable in interpretation to many legitimate uses of "is a part of" in scientific and in technical discourse. This paper has a dual aim: (1) to raise in general the problem of devising an adequate set of axioms for the part relation, which is free from these restrictive features, and (2) to make the constructive step of presenting an axiom system which is at least a first approximation to this desideratum.

More than an academic interest attaches to the problem of constructing such an axiom system. The absence of a formal theory of the part relation able to accommodate a wide portion of the spectrum of scientifically interesting usages of "is a part of"[2] would greatly hamper certain investigations in the philosophy of science. It would, for example, wholly block efforts toward a general clarification of such important concepts as "organic whole" or "system" or "Gestalt."[3] For if we do not possess an exact, formal articulation of the part-whole concept, we are unable fruitfully to subject to precise analysis these other important and far more complex concepts in which the notions of part and whole are inextricably involved. These considerations, though a digression, point clearly to the need for a general and adequate axiomatization of the part relation.

The basis of an axiomatic theory of the part relation is a set D of (otherwise unspecified) "objects"—the fundamental domain or universe of discourse. Let "x," "y," "z" . . . be variables whose range is D. A system of logic which includes elementary set theory is presupposed. To this is added the single primitive relation *Pt* which may be read—and is intended to be interpreted—as "is a part of."

In terms of the primitive *Pt* we may define two further concepts: the relation of discreteness among "objects," and a summation relation between a set of "objects" and the "object" which is the "sum" of this set.

Definition 1. x is discrete from y if they have no part in common:

$$x|y \text{ for } \sim (Ez)\,[zPtx \,\&\, zPty]$$

Definition 2. S has sum x if an "object" is discrete from x if, and only if, it is discrete from every element of S:

$$S \text{ sum } x \text{ for } (z)\,[z|x \equiv (y)\,(y \epsilon S \supset z|y)]$$

Now the mereology of the Polish logicians is based upon the following three axioms, in addition to these definitions:

Axiom 1. If x is a part of y, and y is a part of x, then x and y must be one and the same "object":

$$(x)\,(y)\,[xPty \,\&\, yPtx \supset x = y]$$

Axiom 2. x is a part of y if, and only if, every "object" discrete from y is also discrete from x:

$$(x)\,(y)\,[xPty \equiv (z)\,(z|y \supset z|x]$$[4]

Axiom (schema) 3. If S is a non-empty set of "objects," there exists some "object" which is a sum of this set:

$$(Ex)\,[x \epsilon S] \supset (Ey)\,[S \text{ sum } y]$$

This axiom system entails the following four theorems, which are here stated without proof, the proofs being elementary.

Theorem 1. Every "object" is a part of itself (Pt is reflexive):

$$(x)\,[xPtx]$$

Theorem 2. If x is a part of y, and y is a part of z, then x is a part of z (Pt is transitive):

$$(x)\,(y)\,(z)\,[xPty \,\&\, yPtz \supset xPtz]$$

Theorem 3. An "object" is completely determined by the set of its parts; i.e., for "objects" to be identical, it suffices that they have the same parts (Pt is extensional):

$$(x)\,(y)\,[(z)\,(zPtx \equiv zPty) \supset x = y]$$

Theorem 4. Any two "objects" whatever may be summed; i.e., any pair of "objects" x,y gives rise to an "object" z which is their sum (Universal summability):

$$(x)\,(y)\,(Ez)\,[\{x,y\} \text{ sum } z]$$

Each one of these theorems represents a feature of mereology which curtails its capacity to serve as a formal counterpart of the part relation as this relation functions in actual discourse:

1. Many legitimate senses of "part" are nonreflexive, and do not countenance saying that a whole is a part (in the sense in question) of itself.

The biologists' use of "part" for the functional sub-units of an organism is a case in point.

2. There are various nontransitive senses of "part." In military usage, for example, persons can be parts of small units, and small units parts of larger ones; but persons are never parts of large units. Other examples are given by the various hierarchical uses of "part." A part (i.e., biological sub-unit) of a cell is not said to be a part of the organ of which that cell is a part.

3. The extensionality property, which entails that wholes are the same if they possess the same parts, rules out those senses of "part-whole" in which the organization of the parts, in addition to the mere parts themselves, is involved. Different sentences can consist of the same words.

4. There are senses of "part" which do not qualify the sum or join of two parts as a part in turn. This is the case, for example, with the usage in connection with "machine parts" or "automobile parts." Thus, adequate restrictions must be placed on summability.

On the basis of these considerations, it is plain that mereology possesses serious shortcomings as a general theory of the part relation. How are these shortcomings to be circumvented, and a more satisfactory set of axioms for the part relation obtained?

One possible approach is quite simple and straightforward. Axiom 1 is retained, for it is surely unobjectionable: there is no feasible way in which the condition represented by its antecedent may be met, without satisfying the conclusion. Axiom 2, however, is dropped, and the range of application of the summability axiom, Axiom 3, is restricted to sets of "objects" whose elements fulfill the condition represented by Axiom 2. This also appears to be on safe intuitive grounds.[5] We obtain by this procedure the following set of two axioms:

Axiom I. If x is a part of y, and y is a part of x, then x and y must be one and the same "object":

$$(x)(y)[xPty \,\&\, yPtx \supset x = y]$$

Axiom (schema) II. If S is a non-empty set of "objects," and if each element y of S is such that the "object" x is a part of y if, and only if, every "object" discrete from y is also discrete from x, then there exists an "object" which is a sum of S:

$$(Ex)[x \epsilon S] \,\&\, (y)[y \epsilon S \supset (x)(xPty \equiv (z)(z|y \supset z|x))] \supset (Eu)[S \text{ sum } u]$$

The possibility remains open, that additional axioms might be added to this revised axiom system for the part relation, which would augment its systematic strength without impairing its serviceability for the intended purpose. However, from the construction of this revised axiom set, it is plain that *any particular part concept that satisfies the axioms of mereology will also yield an application or interpretation of the revised axiom system.* On the other hand, it is readily seen that Theorems 1 through 4 do not obtain in the revised axiom system, except under some explicit restrictions upon the "objects" in view. Therefore, *the revised axiom system is not subject to the above-mentioned counter-intuitive restrictions of mereology.* It thus appears that this revised axiom system may provide a step toward an adequate formal theory of the concept "is a part of" as this functions in technical discourse.

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NOTES

[1] S. Lesniewski, "O podstawach matematyki" (in Polish), *Przeglad filozoficzny*, vols. 30–34 (1927–1931); J. H. Woodger, *Axiomatic Method in Biology* (London, 1937); A. Tarski, "Appendix E," *ibid.*; H. S. Leonard and N. Goodman, "The Calculus of Individuals and Its Uses," *Journal of Symbolic Logic*, vol. 5 (1940). A treatise on mereology by B. Sobocinsky is scheduled to appear in the series Studies in Logic and in the Foundations of Mathematics, edited by Brouwer, Beth, and Heyting. The history of efforts at a formal treatment of the part concept dated back to Leibniz in the seventeenth century: N. Rescher, "Leibniz's Interpretation of His Logical Calculi," *Journal of Symbolic Logic*, vol. 19 (1954).

[2] E. Nagel gives an excellent survey of these usages in the article "Wholes, Sums, and Organic Unities," *Philosophical Studies*, vol. 3 (1952).

[3] The present paper was motivated by researches along these lines in which the writer collaborated with Dr. Paul Oppenheim. These will be presented in an article forthcoming in the *British Journal for the Philosophy of Science.*

[4] Instead of taking Pt as primitive, | might have been employed. Then Axiom 2 would be a definition, and Definition 1 an axiom. Some of the expositions of mereology proceed in this way, which is, of course, totally equivalent to that used here.

[5] Due caution requires taking note of the possibility that the summability axiom may perhaps be too strong even in its revised form. This would be the case if some actual and accepted usage of "part" constituted a counter-example. In this event, the restriction on summable sets would require still further strengthening. The writer has, however, been unable to find a usage which necessitates this.

Belief, Synonymity, and Analysis

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In his paper "Synonymity, and the Analysis of Belief Sentences" (*Analysis*, April 1954), Hilary Putnam reports that Carnap remains unconvinced by Church's critique[1] of his attempt to translate belief sentences into an extensional (meta-)language, but is, on the other hand, deeply disturbed